

Chiral symmetry breaking driven by dilaton

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Abstract

Dynamical properties of gauge theories with light flavor quarks are studied in a dual supergravity by adding a D7-brane probe into the AdS background deformed by the dilaton. By estimating the vev of flavor quark-bilinear in both the supersymmetric and non-supersymmetric gravity duals, we find spontaneous chiral symmetry breaking in the case of the non-supersymmetric background. We also study quark-antiquark potential for light quarks to see the quark confinement in the models considered here.

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1 Introduction

It is an interesting problem to make clear the gauge/gravity correspondence from superstring theory [1]. In particular, it is hoped that the correspondence could be applicable to QCD by deforming the anti-de Sitter space-time (AdS) into the non-conformal case. It is however a difficult problem to describe QCD in the ultra-violet (UV) region where we must reproduce the asymptotic freedom and we will then need full string theory to see this property. Then semi-classical gravity would not be sufficient in this region.

Meanwhile near the horizon, in the infrared (IR) region, the semi-classical gravity could provide the characteristic property of QCD vacuum, e.g. quark- confinement and chiral symmetry breaking etc.. Up to now, we could believe that the most plausible vacuum state of QCD is in a gauge field condensate phase [2, 3]. When the vacuum expectation value (vev) of gauge field exists, it could be seen through the dilaton configuration on the gravity side. So it is meaningful to see non-perturbative QCD property in such a gravity background.

Most works to see such a property of gauge theories have been performed for heavy fundamental quarks, while an idea to add light flavor quarks has been recently proposed by Karch and Katz [4] for D3-D7 brane system in $\text{AdS}_5 \times S^5$. Several authors have extended this idea to various 10d gravity backgrounds which have been proposed for various gauge duals, and they have tried to show meson spectrum, chiral symmetry breaking and other related subjects in several appropriate gravity backgrounds [5, 6, 7, 8, 9, 10].

Our purpose here is to study non-perturbative properties of QCD from gravity side through a model, which includes a non-trivial dilaton, by introducing D7-brane probe according to the idea of Karch-Katz. We solve the embedding problem of the D7-brane in some backgrounds to study the chiral symmetry breaking of the flavor quark. We consider firstly a supersymmetric background given in [11, 12]. In this background, there is no singularity, and the quark confinement has been assured for heavy fundamental quark [11, 12]. Here we concentrate on the light flavor quarks to study chiral symmetry breaking and inter-quark potential. Then we extend the same analysis to a non-supersymmetric case.

Quark confinement is seen in both supersymmetric and nonsymmetric cases. While, as for the chiral symmetry, we find that it is not broken in the supersymmetric case, and we need a non-supersymmetric background. Such an example is shown and we assured the spontaneous chiral symmetry breaking (SCSB) for the model given here.

In section 2, we give the setting of our model and the embedding of D7 brane in both supersymmetric and non-symmetric backgrounds to study the breaking of the chiral symmetry. In section 3, the quark-antiquark potential for flavor quarks is studied through the Wilson loop estimation for the two backgrounds, and the summary is given in the final section.

2 Background geometry and embedding of D7 brane

The D7 brane embedding is studied for two types of backgrounds, supersymmetric and non-symmetric one, to see the chiral symmetry breaking for the light flavor quarks introduced through D7 brane probe.

2.1 Supersymmetric background

We consider the $ISO(1,3) \times SO(6)$ symmetric solution given in [11, 12] for 10d IIB model. This solution is supersymmetric and it has no singularity in the bulk, so we can study the dual gauge theory through semi-classical approach of bulk gravity. In the present case, the dual gauge theory for this background preserves $\mathcal{N} = 2$ supersymmetry. The solution can be written in the string frame and taking $\alpha' = g_s = 1$, as follows,

$$ds_{10}^2 = G_{MN} dX^M dX^N = e^{\Phi/2} \left(\frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right), \quad (1)$$

$$e^\Phi = \left(1 + \frac{q}{r^4} \right), \quad \chi = -e^{-\Phi} + \chi_0, \quad (2)$$

where $M, N = 0 \sim 9$, $x^\mu = X^\mu(\mu, \nu = 0 \sim 3)$, $R^4 = 4\pi N g_s$ and q is a constant. Φ and χ denote the dilaton and the axion respectively, and the self-dual five form $F_{\mu_1 \dots \mu_5}$ is given as in [11, 12]. And other field configurations are not used here. This solution connects two asymptotic geometries, $AdS_5 \times S^5$ and flat space-time, respectively, in UV ($r = \infty$) and IR ($r = 0$) limit [11, 12].

We introduce flavor quark by embedding a D7 brane probe which lies in $\{x^\mu, X^4 \sim X^7\}$ directions. Here we rewrite the 6d geometry as $\sum_{M=4}^9 (dX^M)^2 = dr^2 + r^2 d\Omega_5^2 = d\rho^2 + \rho^2 d\Omega_3^2 + (dX^8)^2 + (dX^9)^2$, where $\rho^2 = \sum_{M=4}^7 (X^M)^2$. Then $r^2 = \rho^2 + (X^8)^2 + (X^9)^2$. The brane action for the D7-probe is

$$S_{D7} = -\tau_7 \int d^8 \xi \left(e^{-\Phi} \sqrt{\mathcal{G}} + \frac{1}{8!} \epsilon^{i_1 \dots i_8} A_{i_1 \dots i_8} \right), \quad (3)$$

where $\mathcal{G} = -\det(\mathcal{G}_{ij})$, $i, j = 0 \sim 7$. $\mathcal{G}_{ij} = \partial_{\xi^i} X^M \partial_{\xi^j} X^N G_{MN}$ and τ_7 represent the induced metric and the tension of D7 brane respectively. Here we consider the case of zero $U(1)$ gauge field on the brane, but we notice that the eight form potential $A_{i_1 \dots i_8}$, which is Hodge dual to the axion, couples to the D7 brane minimally. We obtain the eight form potential $A_{(8)}$ as $F_{(9)} = dA_{(8)}$ in terms of the Hodge dual field strength $F_{(9)}$ [13]. By taking the canonical gauge, $\xi^i = X^i$, we fix the embedding by solving the equation of motion for the fields $X^8(\xi)$ and $X^9(\xi)$ under the ansatz, $X^9 \equiv w(\rho)$ and $X^8 = 0$, without loss of generality. Then the induced metric and the action (3) are reduced as

$$ds_8^2 = e^{\Phi/2} \left(\frac{r^2}{R^2} dx^\alpha dx_\alpha + \frac{1 + (w')^2}{r^2} R^2 d\rho^2 + \frac{\rho^2}{r^2} R^2 d\Omega_3^2 \right) \quad (4)$$

$$S_{D7-S} = -\tau_7 \int d^8 \xi \sqrt{\epsilon_3} \rho^3 \left(1 + \frac{q}{r^4} \right) \left(\sqrt{1 + (w')^2} - 1 \right), \quad (5)$$

where $w' = dw/d\rho$ and ϵ_3 is the determinant of three sphere. And we obtain the following equation

$$w'' + \frac{3}{\rho}w'(1 + (w')^2) + \frac{4q}{r^2(r^4 + q)} \left\{ (w - \rho w')(1 + (w')^2) - w(1 + (w')^2)^{3/2} \right\} = 0, \quad (6)$$

The solution w determines the embedding of the D7-brane, and the problem of the chiral symmetry breaking is also solved from the viewpoint of the dual gauge theory. The latter point is understood from the asymptotic form of the solution. For $r \rightarrow \infty$ (i.e. $\rho \rightarrow \infty$), the solution $w(\rho)$ can be solved as

$$w(\rho) \sim m + \frac{c}{\rho^2} \sim m + \frac{c}{r^2}. \quad (7)$$

Since the conformal dimension of w is three [4], then we can interpret m and c are quark mass and the vev of quark bilinear $\langle \bar{\psi}\psi \rangle$, respectively, from the gravity/gauge correspondence.

In the same holographic context, we can give a comment on the form of the dilaton e^Φ . It represents the running coupling of the dual gauge theory, and the parameter q can be interpreted as the gauge field condensate, $\langle F_{\mu\nu}F^{\mu\nu} \rangle$. As for the relation between q and $\langle F_{\mu\nu}F^{\mu\nu} \rangle$, further consideration from gauge theory side is given in [12]. So we abbreviate them, but this parameter is the most important factor of the present model.

Equation (6) yields constant solutions, $w(\rho) = m$, and non-constant ones with $c \neq 0$ for each m . The latter solutions are obtained numerically and are shown in the Fig.1. But they should be abandoned by the two reasons; (i) their energies are always higher than that of the constant solution with the same m , and (ii) they can not be interpreted from the AdS/CFT context due to the lack of one-to-one correspondence of ρ and r [7]. On the other hand, all the constant solutions are equally possible since their energies are degenerate to zero, $-S_{D7-S} = 0$. Then we can say that SCSB does not occur in the supersymmetric background.

This result is reasonable. For supersymmetric background, there is no force between D3 and D7 branes for $w' = 0$, then the value of $w(\rho)$ can not be changed from $w(\infty) = m_q$ when $\rho \rightarrow 0$, and $w(\rho) = m_q$ is preserved up to $\rho = 0$. Therefore, in order to realize SCSB, it would be necessary to consider a non-supersymmetric background solution as shown in the next subsection.

Before considering the non-supersymmetric background, we comment on the supersymmetry for the solutions of $w' \neq 0$. The background (1) has 1/2 supersymmetry of IIB theory and its Killing spinor is given as [11]

$$\epsilon = e^{\Phi/4} \left(\frac{r^2}{R^2} \right)^{1/4} \epsilon_0, \quad (8)$$

where ϵ_0 is a constant spinor which satisfies $i\sigma_2 \otimes \Gamma_{0123}\epsilon_0 = \epsilon_0$. Here Γ_{0123} denotes the antisymmetrized product of Γ_A which are 10d flat space Γ matrices. After embedding D7-brane probe with the above solution for $w(\rho)$, the supersymmetry is in general broken except for constant w . This is easily seen as follows. When some supersymmetry

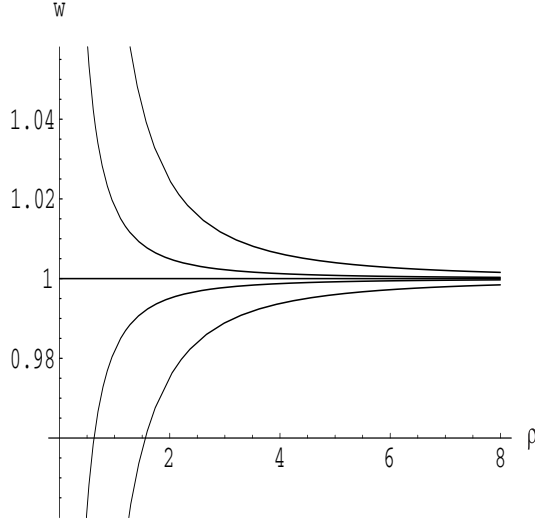


Fig. 1: The solution w as a function of ρ . Here we set $q = 1$. The solutions are depicted for five cases of $c = -0.1, -0.02, 0, 0.02, 0.1$ with $m = 1$ fixed. When c is finite, the solutions are divergent at $\rho = 0$. The regular solution is the mass only solution $w(\rho) = m$ represented by the horizontal line.

remains, the above Killing spinor must satisfy the following condition [17] due to the κ symmetry of D7 brane,

$$\Gamma\epsilon = \epsilon, \quad \Gamma = i\sigma_2 \otimes \Gamma_{(0)} \quad (9)$$

$$\Gamma_{(0)} = \frac{1}{8!\sqrt{\mathcal{G}}} \epsilon^{i_1 \dots i_8} \partial_{i_1} X^{M_1} \dots \partial_{i_8} X^{M_8} \Gamma'_{M_1 \dots M_8} \quad (10)$$

where $\Gamma'_{M_1 \dots M_8}$ is the totally antisymmetrized product of $\Gamma'_M = E_M^A \Gamma_A$. In our case, in the Cartesian coordinate, we obtain

$$\Gamma_{(0)} = \frac{1}{\prod_{i=4}^7 \sqrt{1 + (X^i w'/\rho)^2}} \left(\Gamma_{(1)} + \frac{w'}{\rho} \sum_4^7 X^i \Gamma_{(i)} \right) \quad (11)$$

where $\Gamma_{(1)} = \Gamma_{01234567}$ and $\Gamma_{(i)} (7 \geq i \geq 4)$ is the one in which the number i is replaced by 9 in $\Gamma_{(1)}$. When $w' = 0$, the condition of the supersymmetry is written as

$$i\sigma_2 \otimes \Gamma_{(1)} \epsilon_0 = \epsilon_0 \quad (12)$$

and we could find 1/2 supersymmetry of the original one. However, for non-zero w' we find that supersymmetry is completely broken. So it is natural to consider a non-supersymmetric background to see SCSB.

2.2 Non-supersymmetric background

Here we consider a non-supersymmetric solution [14, 15, 16] which is given without changing the five form field and eliminating the axion, $\chi = 0$, as,

$$ds_{10}^2 = e^{\Phi/2} \left(\frac{r^2}{R^2} A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right), \quad (13)$$

$$A(r) = \left(1 - \left(\frac{r_0}{r}\right)^8\right)^{1/4}, \quad e^\Phi = \left(\frac{(r/r_0)^4 + 1}{(r/r_0)^4 - 1}\right)^{\sqrt{3/2}}. \quad (14)$$

This configuration has a singularity at the horizon $r = r_0$, and the present semi-classical analysis can not be applied near this point. So we avoid this point in the followings.

As for the the dilaton, e^Φ can be expanded as

$$e^\Phi \sim 1 + \frac{q_{\text{NS}}}{r^4}, \quad q_{\text{NS}} = \sqrt{6}r_0^4. \quad (15)$$

As shown in the previous case, in the context of AdS/CFT, it would be possible to interpret the parameter q_{NS} as the gauge-field condensate $\langle F_{\mu\nu}F^{\mu\nu} \rangle$.

The D7 brane is embedded as above, and the following brane action is obtained,

$$\begin{aligned} S_{\text{D7-NS}} &= -\tau_7 \int d^8\xi \mathcal{L}_{\text{NS}} \\ &= -\tau_7 \int d^8\xi \sqrt{\epsilon_3} \rho^3 \left(\frac{(r/r_0)^4 + 1}{(r/r_0)^4 - 1} \right)^{\sqrt{3/2}} \left(1 - \left(\frac{r_0}{r}\right)^8 \right) \sqrt{1 + (w')^2}. \end{aligned} \quad (16)$$

We could expect to find SCSB by solving the equation of motion for w ,

$$\partial_w \mathcal{L}_{\text{NS}} - \partial_\rho \frac{\partial \mathcal{L}_{\text{NS}}}{\partial w'} = 0. \quad (17)$$

We consider the neighborhood of $\rho = 0$ and set $w(0)' = 0$ since the solution must

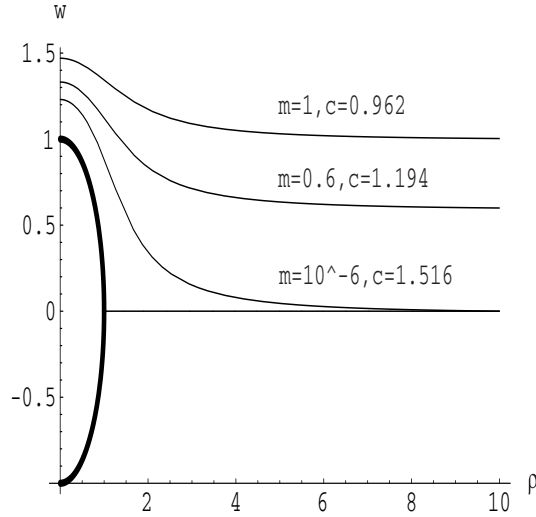


Fig. 2: The solution w as a function of ρ . Here, we take $r_0 = 1$, or $q_{\text{NS}} = \sqrt{6}$. The non-trivial solutions are depicted for three cases of $m = 10^{-6}$, 0.6 , 1 . The curve for $m = 10^{-6}$ agrees with the one in the chiral limit within the thickness of line. The thick-solid curve represents the horizon $w^2 + \rho^2 = r_0^2$. The horizontal line shows the trivial solution ($w(\rho) = 0$) and the endpoint touches the horizon at $(\rho, w) = (1, 0)$.

be an even function of ρ . Then Eq. (17) is solved numerically for $w(\rho)$ by setting $w(0)' = 0$ and $w(0) > r_0$; note that there exists a singularity at $r = \sqrt{\rho^2 + w^2} = r_0$. Figure 2 shows the solutions for three values of $w(0)$, where $r_0 = 1$, or $q_{\text{NS}} = \sqrt{6}$, is taken. Each solution yields a different set of m and c . As for the solution of the $m = 0$ limit, c is finite. Thus, the spontaneous chiral symmetry breaking takes place in the present non-supersymmetric background. Other two solutions also show finite $\langle \bar{\psi}\psi \rangle$. The solutions become flatter, as m increases. Similar ρ dependence of $w(\rho)$ is also seen in [7] for a D7 insertion in the Constable-Myers background [18] which is non-supersymmetric and has a non-constant dilation. The solutions which touch on the horizon are omitted here by the reason mentioned above. A typical example of such solutions is the trivial solution $w(\rho) = 0$, as shown in Fig. 2. So the trivial solution, $w = 0$, is abandoned here differently from the supersymmetric case.

3 Quark-antiquark potential

In this section, we study the static potential between a dynamical quark-antiquark pair in the above two backgrounds.

3.1 Supersymmetric background

In the case of $\mathcal{N} = 2$ symmetric and constant Φ , the potential has been estimated in [5]. And it has been shown that non-confinement Coulomb potential is seen at large separation of the quarks. Here we show, for finite quark mass, the linear rising potential which has been shown for the case of infinitely heavy quarks [11, 12].

Here we consider the supersymmetric case given above. The relevant part of the AdS_5 metric is written as

$$ds^2 = e^{\Phi/2} \left(\frac{r^2}{R^2} (-dt^2 + dx^2) + \frac{R^2}{r^2} dr^2 \right). \quad (18)$$

We consider a string whose endpoints lie on D7-brane at a distance $2L$ from one another. And the string is straddling the point $x = 0$ about which the profile is symmetric. Then the string action per unit time, namely its energy, is given as

$$E = \frac{1}{\pi} \int_0^L dx \, n \sqrt{\frac{r^4}{\lambda} + (r')^2}, \quad n = \sqrt{1 + \frac{q}{r^4}}, \quad (19)$$

where $\lambda = R^4$ denotes the 'tHooft coupling and $r' = dr/dx$; here we take $g_s = 1$ for simplicity. Since the "Lagrangian" is independent of x , we can introduce a constant u_0 as

$$\sqrt{\lambda} u_0^2 = \frac{n}{\sqrt{\frac{r^4}{\lambda} + (r')^2}} r^4. \quad (20)$$

From this, we obtain the following by introducing $y = r/u_0$,

$$L = \frac{\sqrt{\lambda}}{u_0} \int_{y_{\min}}^{y_{\max}} \frac{dy}{y^2} \frac{1}{\sqrt{\tilde{q} + y^4 - 1}}, \quad \tilde{q} = \frac{q}{u_0^4}, \quad (21)$$

where

$$y_{\max} = r_{\max}/u_0, \quad y_{\min} = (1 - \tilde{q})^{1/4}. \quad (22)$$

Here we comment on the above upper bound r_{\max} . We should take it as $r_{\max} = w(0) > 0$, however $w(0) = w(\infty)$ in the present case since $w(\rho)$ is a constant. Then we can consider as $r_{\max} = 2\pi m_q$ as in the AdS case of $q = 0$.

As for the energy, we obtain

$$E = \frac{u_0}{\pi} \int_{y_{\min}}^{y_{\max}} \frac{dy}{y^2} \frac{\tilde{q} + y^4}{\sqrt{\tilde{q} + y^4 - 1}}. \quad (23)$$

For $q = 0$, the above formula are equivalent to the one given in [5], and we find the Coulomb potential at long distance in this case. This means that two quarks can be separated by very small energy at infinitely long distance, $L \rightarrow \infty$, where the energy approaches to $2m_q$ as shown in Fig.3. Then quarks are not confined and could be free.

For the case of $q > 0$, the lower bound in the above integrals is $(1 - \tilde{q})^{1/4}$ and the dominant contribution is obtained from the region of $\tilde{q} = q/u_0^4 \sim 1$ or $u_0 \sim q^{1/4}$. Near this region, we obtain large L , and we can estimate the above formulas as

$$L = \left(\frac{\lambda^2}{q}\right)^{1/4} \frac{1}{3y_{\min}^3} \quad (24)$$

$$E = 2m_q + \frac{\sqrt{q/\lambda}}{\pi} L \quad (25)$$

This result expresses the linear potential and the effective string tension is given as

$$\tau = \frac{\sqrt{q/\lambda}}{\pi} \quad (26)$$

which is equivalent to the one given for the case of heavy quarks [12]. So the energy of the bound state, in this confinement case, exceeds $2m_q$ at some finite value of L , and increases with L infinitely. However, in the present case, there are light dynamical quarks, then the bound state would decay to two mesons by a pair creation of quark and antiquark. The total energy of newly created two mesons would be small when the distance between quark and antiquark in those mesons is small enough. So the transition from large L bound state to light two mesons would be realized.

In the region of small L , which is realized for large u_0 , we could observe the similar potential to the case of $q = 0$ given in [5]. This is easily understood by considering the fact that the above formula for L and E can be approximated by the one of $q = 0$ for $q/u_0 \ll 1$. We notice that we observe a linear potential again at very small L with an effective string tension as given in [5]. The tension is proportional to m_q^2 and is different from the one given above for large L . This linear potential observed at small L is caused by the strong coupling gauge theory in the UV limit. This is then different from the case of real QCD which is asymptotic free. The typical potential profile mentioned above is shown in the Fig.3 where two cases of $q = 5$ and $q = 0$ are shown.

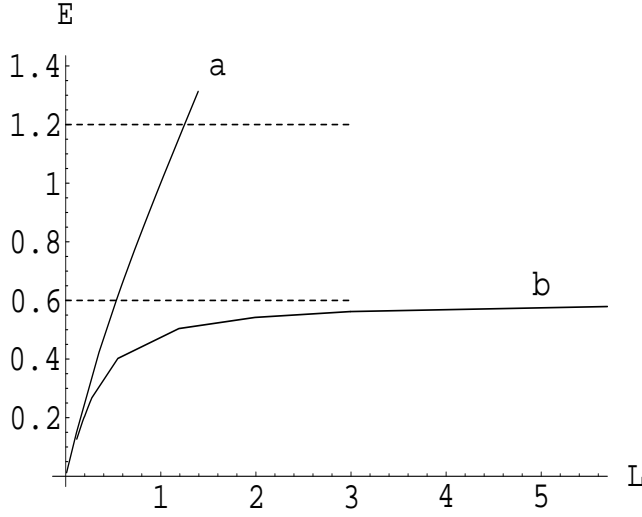


Fig. 3: The energy E of the string versus quark-antiquark distances $2L$ are shown. The solid curve represents the one for (a) $q = 5$, the confinement case, and for (b) $q = 0$, non-confinement case. The normalization is free and we set as $m_q = 0.3$, $g_s = 1$.

Finally, we comment on the case of electric condensate, $q < 0$. The integrant of the Equation (19) is rewritten as, $\tilde{n}\sqrt{1/\lambda + (r')^2/r^4}$ and $\tilde{n} = \sqrt{q + r^4}$, and \tilde{n} has no finite minimum for $q < 0$. Then the potential between quark-antiquark does not show linear potential. It is easy to see this by numerical estimation of E as a function of L . As shown in the previous section, instability appears in this case, and a peculiar behavior will be seen also in the calculation of the Wilson loop. So we need a care to study this quantity. We will discuss on this point in elsewhere.

3.2 Non-supersymmetric case

Next, we consider the non-supersymmetric solution, (13) and (14). After the same procedure as the supersymmetric case, we obtain the distance $2L_{\text{NS}}$ and the energy E_{NS} for the light quark and anti-quark in terms of an integral constant H ,

$$H = e^{\Phi/2} \left(\frac{r}{R}\right)^4 \frac{A(r)^3}{\sqrt{\left(\frac{r}{R}\right)^4 A(r)^2 + (r')^2}}, \quad (27)$$

given as follows

$$L_{\text{NS}} = \int_{r_{\min}}^{r_{\max}} \frac{dr}{r^2} \frac{R^2}{A(r)\sqrt{F(r) - 1}}, \quad F(r) = \frac{e^{\Phi}}{H^2} A(r)^4 \left(\frac{r}{R}\right)^4, \quad (28)$$

$$E_{\text{NS}} = \frac{1}{\pi} \int_{r_{\min}}^{r_{\max}} \frac{dr}{R^2} \frac{e^{\Phi} A(r)^3 r^2}{H \sqrt{F(r) - 1}}. \quad (29)$$

Here Φ and $A(r)$ are given in (13) and (14), and the lower bound r_{\min} of the integration is given by

$$F(r_{\min}) = 1. \quad (30)$$

This is the middle point of the string connecting quark and anti-quark, and $r' = dr/dx$ should be zero at this point due to the smoothness of the string configuration. Actually we can see the equivalence of the equations $F(r_{\min}) = 1$ and $r'(r_{\min}) = 0$.

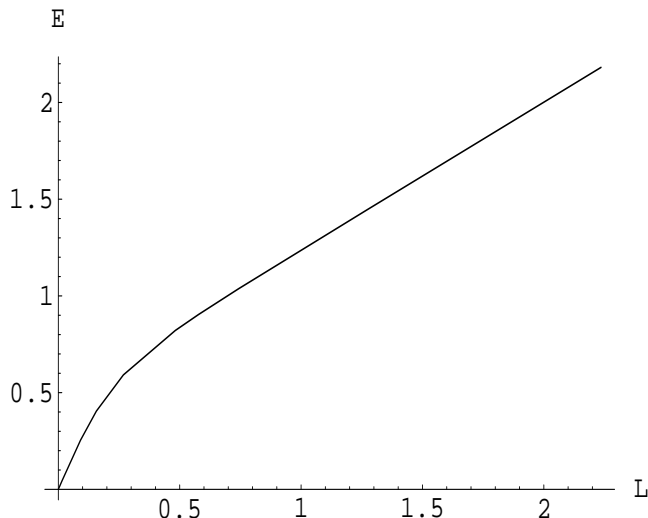


Fig. 4: The energy of the string versus quark-antiquark distance is shown for non-supersymmetric case, (29) and (28). Here we set $m_q = 3/(2\pi)$, $g_s = 1$, $r_0 = 1$ and $R = 1$.

From the above formula, we can observe the linear potential for large L as pointed out by [14]. In Fig.4, an example of the numerical estimation for the above E and L is shown.

4 Summary

A supersymmetric and a non-supersymmetric background deformed by the dilaton are studied in the context of gauge/gravity correspondence by embedding a D7 brane. The supersymmetric background considered here is dual to the $\mathcal{N} = 2$ supersymmetric Yang-Mills theory with gauge field condensate. In this case, we find the energy minimum configuration of the embedded D7 brane which is a flat plane in the eight dimensional space-time. This solution implies zero vev of flavor quark bilinear $\langle \bar{\psi}\psi \rangle$, then the chiral symmetry is preserved. We find other non-flat solutions with $\langle \bar{\psi}\psi \rangle \neq 0$. However, these solutions break supersymmetry and have higher energies than the one of the supersymmetric flat solution for any quark-mass case. Then we can say that the chiral symmetry is preserved in this background.

And, for any finite quark mass, we find the quark confinement in this case by estimating the Wilson-loop. Namely, we could find a linear potential for large separation of

quark and anti-quark. For the case of electric gauge-field condensate, the background has a singular point where the action of the embedded D7 changes its sign. This implies that the available region for AdS/CFT analysis is bounded by the magnitude of the electric field condensate, and we couldn't find the quark confinement in this state.

As a result, in order to find a chiral symmetry breaking, we must consider non-supersymmetric background or an D7 embedding which breaks supersymmetry. Here we examined a non-supersymmetric background of IIB model, and we could find a chiral symmetry breaking. And the inter-quark potential given by the Wilson loop shows the confining force. However, in our non-supersymmetric background configuration, there is a singular point. So we must make the analysis out of this singularity. One should notice that both results, CSB and the quark confinement, are obtained out of the singular region.

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